LAST Name: ANSWER.

First Name: _____

Student Number: _____

MIE 200F - Quiz number 1b - September 19, 2002
quiz duration = 20 minutes

A turkey is running at a constant speed of 5 m/s in a straight line in the y-direction, as shown.
At time t = 0, it is located at the point (x,y) = (-5, 2).

(4) (a) Find v_θ of the turkey at t = 0, if θ is measured with respect to the x-axis, as shown.

(2) (b) Find the radius of curvature ρ of the turkey's path at t = 0.

(4) (c) Consider the following three variables: $\dot{r}$, $\ddot{\theta}$, a_θ. Which of them is greater than zero at t = 0? Explain briefly.

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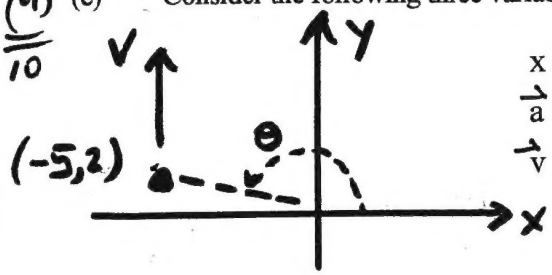


$x = x_o + v_o t + \frac{1}{2} a_o t^2$ $\qquad v = v_o + a_o t$

$\vec{a} = \dot{v}\,\hat{e}_t + v\dot{\theta}\,\hat{e}_n = \dot{v}\,\hat{e}_t + v^2/\rho\,\hat{e}_n$ $\qquad a\,dx = v\,dv$

$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$ $\qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$

(a) $\theta = \tan^{-1}\left(\frac{2}{-5}\right)$
$= 158.2^\circ$
or $158.2 \times \frac{\pi}{180} = 2.76$ radians

$\hat{e}_\theta$ is in direction $158.2 + 90 = 248.2^\circ$

$\vec{V}$ is in the $\hat{j}$ direction

$V_\theta = 5\ m/s\ \cos(248.2 - 90)$
$= -4.64\ m/s$

(b) ρ is infinity (or undefined)

(c) $\dot{r} > 0$, as turkey is moving away from origin.
$\ddot{\theta} > 0$, as $\dot{\theta}$ is negative with its magnitude decreasing towards 0.

$a_\theta = 0$, because $\vec{a} = 0$

Last Name ANSWER.

First Name _____

Student number _____

MIE200F- Quiz number 1b- September 18, 2003

Quiz duration: 20 minutes

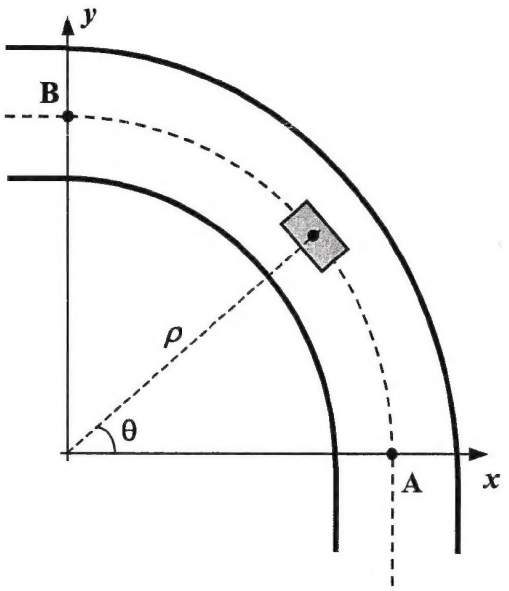
In the shown figure, the car driver enters the curved road at point **A** with a speed of 120 km/hr while decelerating at a constant rate of 4 m/s^2. Assuming the radius of curvature ρ of the road to be constant and equal to 80m:

(i) evaluate the car speed at θ=45°,

(ii) write the vector expression of the acceleration in *n-t* coordinates system at point **B**.

Useful formulas: $\vec{a} = \frac{v^2}{\rho}\hat{e}_n + v\hat{e}_t$, $v^2 = v_o^2 + 2as$

Solution



(i) at θ= 45°

$v^2 = v_o^2 + 2as$

v_o= 120/3.6= 33.34 m/s

s=π/4*80=62.83m

$$v = \sqrt{v_o^2 + 2as} = \sqrt{33.34^2 - 2*4*62.83} = 24.78 \text{ m/s}$$

(ii) at θ= 90°

$v^2 = v_o^2 + 2as$

s=π/2*80=125.66 m

$$v = \sqrt{v_o^2 + 2as} = \sqrt{33.34^2 - 2*4*125.66} = 10.31 \text{ m/s}$$

$$\vec{a} = \frac{v^2}{\rho}\hat{e}_n + v\hat{e}_t = \frac{10.31^2}{80}\hat{e}_n - 4\hat{e}_t = 1.33\hat{e}_n - 4\hat{e}_t \text{ m/s}^2$$

Midterm MIE 200 1999

Question 2: (15 marks)

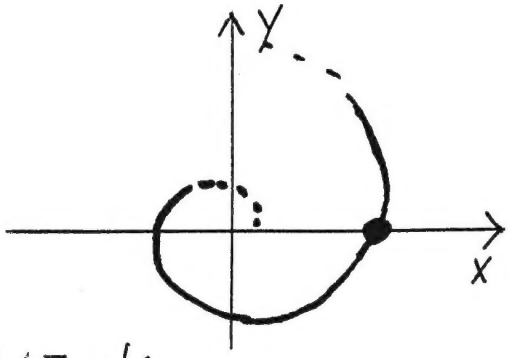
A woman is walking on a spiral path as shown in the diagram. The *magnitude* of her velocity (her speed) is constant and equal to 2 m/s. The distance between her and the center of the spiral is increasing at a constant rate of 1.1 m/s. At the position shown in the diagram, she is 5 meters from the center of the spiral, at the location (x,y) = (5,0).

3 (a) At the position shown, what are the values of $\dot{r}$ and $\ddot{r}$?

4 (b) At the position shown, what is the velocity expressed in the x-y coordinate system?

4 (c) At the position shown, what is the value of $\dot{\theta}$?

4 (d) At the position shown, what is the value of $\ddot{\theta}$?



(top view)

15/15

(a) From wording of question,
$\dot{r} = 1.1$ m/s, $\ddot{r} = 0$

(b) $|V_r| = 1.1$

$V_\theta = \sqrt{V^2 - V_r^2} = \sqrt{2^2 - 1.1^2} = 1.67$ m/s

$\Rightarrow \vec{V} = (1.1\,\hat{i} + 1.67\,\hat{j})$ m/s.

(c) $|\vec{V}_\theta| = 1.67 = r\dot{\theta} = 5\dot{\theta}$

$\Rightarrow \dot{\theta} = 1.67/5 = 0.334\ s^{-1}$

(d) $V_r^2 + V_\theta^2 = |V|^2$

$(1.1)^2 + V_\theta^2 = 2^2$

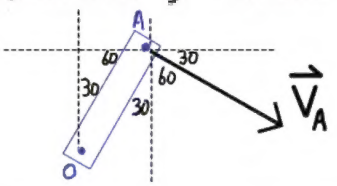
$\Rightarrow V_\theta = 1.67 = \text{constant} = r\dot{\theta}$

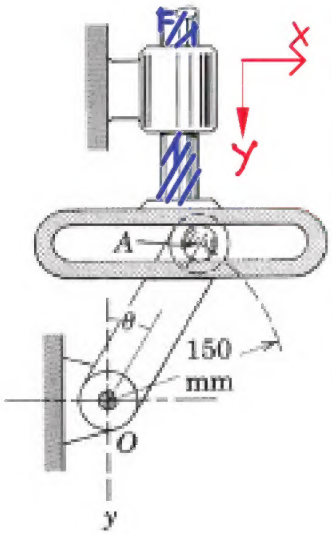
$\frac{d}{dt}\{ r\dot{\theta} = 1.67 \} \Rightarrow \dot{r}\dot{\theta} + r\ddot{\theta} = 0$

$\Rightarrow \ddot{\theta} = -\dot{r}\dot{\theta}/r = (-1.1)(.334)/5 =$

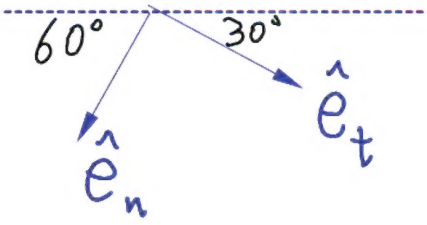
$\ddot{\theta} = -0.0735\ s^{-2}$

Rotation of crank OA is controlled by the horizontal slotted link which has a downward acceleration and velocity of $3.5\ m/s^2$ and $0.3\ m/s$, respectively, when $\theta = 30°$. For this instant determine the velocity and acceleration of A. Also find the corresponding values of $\dot{\theta}$ and $\ddot{\theta}$.

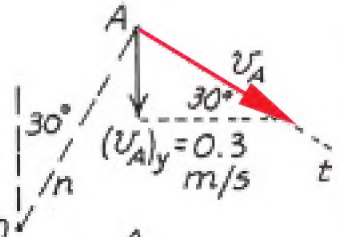




$\vec{V}_A \perp \overline{OA}$



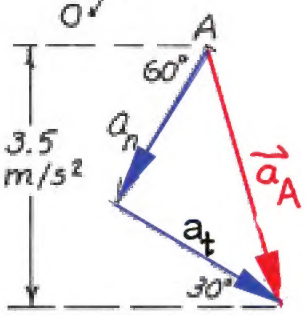
Point A has circular motion so has tangential velocity and n- and t- components of acceleration.



$v_A = (v_A)_y / \sin 30° = \frac{0.3}{0.5} = \boxed{0.6\ m/s}$

$v_A = r\dot{\theta}$, $\dot{\theta} = 0.6/0.150 = \boxed{4\ rad/s}$

$a_n = v_A^2/r = 0.6^2/0.150 = 2.4\ m/s^2$



Thus $2.4 \sin 60° + a_t \sin 30° = 3.5$

$a_t = 2.84\ m/s^2$, $a_A = \sqrt{2.4^2 + 2.84^2} = \boxed{3.72\ m/s^2}$

$a_t = r\ddot{\theta}$, $\ddot{\theta} = 2.84/0.150 = \boxed{18.95\ rad/s^2}$

JLM

Recall: For <u>circular</u> motion:

$a_t \sim a_\theta$

$a_n \sim -a_r$

r-θ: $\dot{r} = \ddot{r} = 0$

$$\vec{a} = -r\dot{\theta}^2 \hat{e}_r + r\ddot{\theta}\hat{e}_\theta$$

n-t:

$$\vec{a} = v^2/\rho\ \hat{e}_n + \dot{v}\hat{e}_t$$

LAST Name: _____ First Name: _____ Student #: _____

MIE100S: Dynamics winter 2013: Quiz A
quiz duration – 25 minutes

At time $t = 0$, a particle is moving counterclockwise in a circle of radius 6 meters around the origin, and slowing down at a rate of 0.35 m/s^2. At time $t = 0$, the particle is located at (r, θ) = (6 meters, 2 radians), and has a speed of 5 m/s.

4 (a) At time $t = 0$, express the velocity in the given x-y rectangular coordinate system.
3 (b) At time $t = 0$, determine $\dot{\theta}$ in S.I. units.
3 (c) At time $t = 0$, determine $\ddot{\theta}$ in S.I. units.
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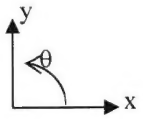


Possibly useful equation:

$x = x_o + v_o t + \frac{1}{2} a_o t^2 \qquad v = v_o + a_o t \qquad a\,dx = v\,dv \qquad \vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$

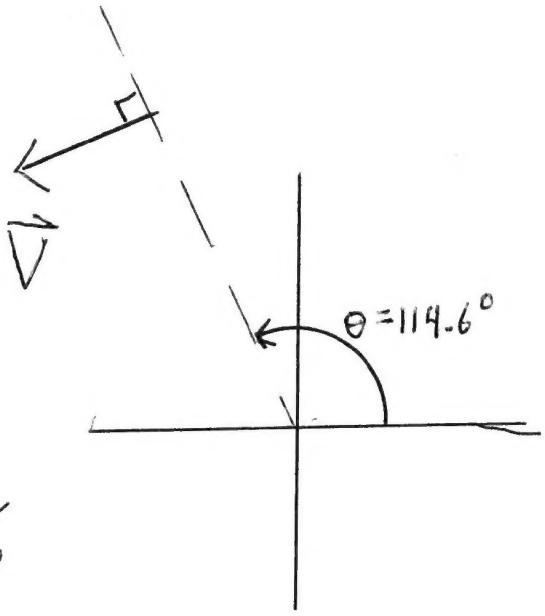
$\vec{a} = \dot{v}\,\hat{u}_t + v\dot{\theta}\hat{u}_n = \dot{v}\,\hat{u}_t + v^2/\rho\hat{u}_n \qquad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{u}_\theta$

(a) 2 radians $= (2)\left(\frac{180}{\pi}\right) = 114.6^\circ$

$V_x = 5\cos(114.6^\circ + 90^\circ) = -4.54\ m/s$

$V_y = 5\sin(114.6^\circ + 90^\circ) = -2.08\ m/s = 5\cos(114.6^\circ)$

$\vec{V} = -4.54\,\hat{i} - 2.08\,\hat{j}\ m/s$



(b) $V_\theta = r\dot{\theta} \Rightarrow \dot{\theta} = \frac{V_\theta}{r} = \frac{V}{r} = \frac{5}{6}$

$\dot{\theta} = 0.83\,s^{-1}$

(c) For a circle, $a_\theta = r\ddot{\theta} = -0.35\ m/s^2$

$\Rightarrow \ddot{\theta} = \frac{a_\theta}{r} = \frac{-0.35}{6} = -0.0583\,s^{-2}$